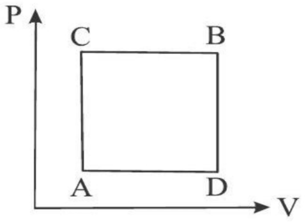
Thermodynamics

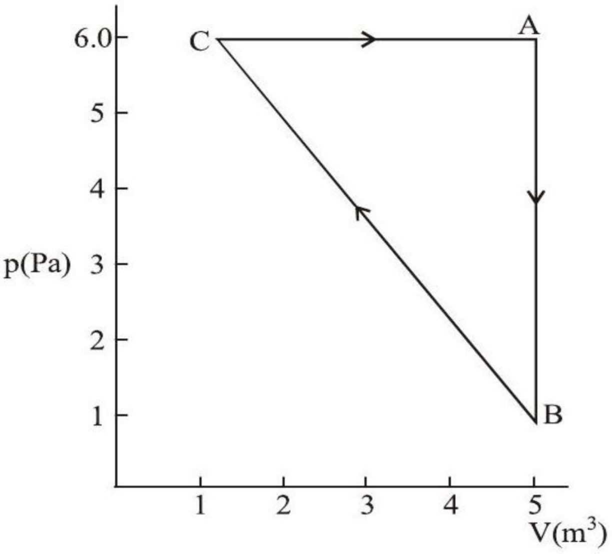
1. A gas can be taken from A to B via two different processes ACB and ADB.



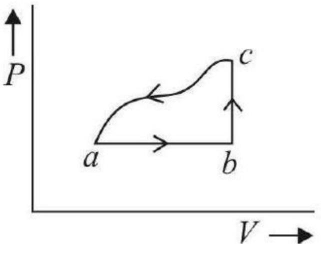
When path ACB is used 60 J of heat flows into the system and 30 J of work is done by the system. If path ADB is used work done by the system is 10 J. The heat (in joule) flow into the system in path ADB is :

2. Two Carnot engines A and B are operated in series. The first one, A receives heat at $T_1 (= 600\ K)$ and rejects to a reservoir at temperature T_2. The second engine B receives heat rejected by the first engine and in turn, rejects to a heat reservoir at $T_3 (= 400\ K)$. Calculate the temperature T_2 in kelvin if the work outputs of the two engines are equal:

3. For the given cyclic process CAB as shown for gas, the work done (in joule) is:

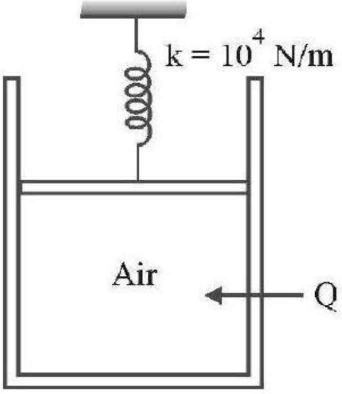


4. A sample of an ideal gas is taken through the cyclic process abca as shown in the figure. The change in the internal energy of the gas along the path ca is - 180 J, The gas absorbs 250 J of heat along the path ab and 60 J along the path bc. The work done (in joule) by the gas along the path abc is :

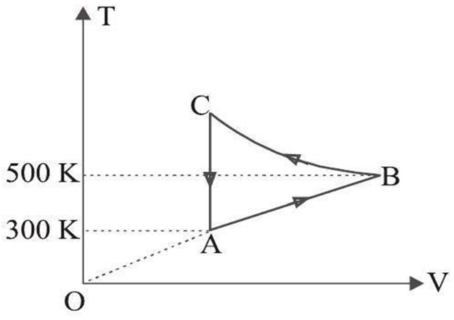


5. 1.0 m^3 of water is converted into 1671 m^3 of steam at atmospheric pressure and 100°C temperature. The latent heat of vapourisation of water is 2.3×10^6 J/kg. If 2.0 kg of water be converted into steam at atmospheric pressure and 100°C temperature, then how much will be the increase in its internal energy (in joule)? Density of water is 1.0×10^3 kg/m^3, atmospheric pressure = 1.01×10^5 N/m^2.

6. Air is contained in a piston - cylinder arrangement as shown in Fig. with a cross - sectional area of 4 cm^2 and an initial volume of 20 cc. The air is initially at a pressure of 1 atm and temperature of 20°C. The piston is connected to a spring whose spring constant is $k = 10^4$ N/m, and the spring is initially undeformed. How much heat (in joule) must be added to the air to increase the pressure to 3 atm. (For air, $C_V = 718$ J/kg°C, molecular mass of air 28.97)



7. Consider the cyclic process $ABCA$, shown in Fig. performed on a sample of 2.0 mole of an ideal gas. A total of 1200 J of heat is withdrawn from the sample in the process. Find the work done (in joule) by the gas during the part BC.



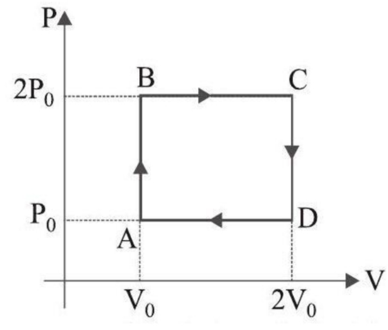
8. A motor car tyre has a pressure of 2 atmosphere at room temperature of 27°C. If the tyre suddenly bursts, find the resulting temperature (in kelvin).

9. A Carnot engine whose heat sink is at 27°C has an efficiency of 40%. By how many degrees should the temperature of the source be changed to increase the efficiency by 10% of the original efficiency?

10. Five moles of an ideal gas taken in a Carnot engine working between 100°C and 30°C. The useful work done in one cycle is 420 joule. If the ratio of the volume of the gas at the end and beginning of the isothermal expansion is $\frac{115}{x}$. Find the value of x.
(Take $R = 8.4$ J/mol. K)

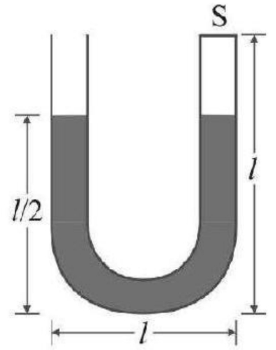
11. How much energy in watt hour may be required to convert 2 kg of water into ice at $0°C$, assuming that the refrigerator is ideal? Given temperature of freezer is $-15°C$, room temperature is $25°C$ and initial temperature of water is $25°C$.

12. Helium gas goes through a cycle $ABCDA$ (consisting of two isochoric and isobaric lines) as shown in figure. Find efficiency of this cycle. (Assume the gas to be close to an ideal gas)



13. Find the amount of work done (in joule) to increase the temperature of one mole of an ideal gas by $30°C$ if it is expanding under the condition $V \propto T^{2/3}$. ($R = 1.99 cal/mol - K$)

14. A thin U-tube sealed at one end consists of three bends of length $l = 250$ mm each, forming right angles. The vertical parts of the tube are filled with mercury to half the height.

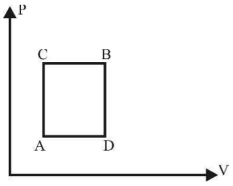


All of mercury can be displaced from the tube by heating slowly the gas in the sealed end of the tube, which is separated from the atmospheric air by mercury. Determine the work done (in joule) by the gas there by if the atmospheric pressure is $P_0 = 10^5$ Pa, the density of mercury is $\rho_{mer} = 13.6 \times 10^3\ kg/m^3$, and the cross-sectional area of the tube is $S = 1\ cm^2$.

15. A Carnot engine having a perfect gas as the working substance is driven backward and is used for freezing water already at $0°C$. If the engine is driven by a 500 W electric motor having a efficiency of 60%, how long (in seconds) will it take to freeze 15 kg of water. Take $15°C$ and $0°C$ as the working temperatures of the engine and assume there are no heat losses in the refrigerating system. Latent heat of ice $= 333 \times 10^3$ J/kg.

SOLUTIONS

1. **(40)**



ΔU remains same for both paths ACB and ADB

$\Delta Q_{ACB} = \Delta W_{ACB} + \Delta U_{ACB}$

$\Rightarrow 60\ J = 30\ J + \Delta U_{ACB}$

$\Rightarrow U_{ACB} = 30\ J$

$\therefore\ \Delta U_{ADB} = \Delta U_{ACB} = 30\ J$

$\Delta Q_{ADB} = \Delta U_{ADB} + \Delta W_{ADB}$

$= 10\ J + 30\ J = 40\ J$

2. **(500)** $\eta_A = \frac{T_1 - T_2}{T_l} = \frac{w_A}{Q_1}$

and, $\eta_B = \frac{T_2 - T_3}{T_2} = \frac{W_B}{Q_2}$

According to question,

$W_A = W_B$

$\therefore\ \frac{Q_1}{Q_2} = \frac{T_1}{T_2} \times \frac{T_2 - T_3}{T_1 - T_2} = \frac{T_1}{T_2}$

$\therefore\ T_2 = \frac{T_l + T_3}{2}$

$= \frac{600 + 400}{2}$

$= 500K$

3. **(10)** Total work done by the gas during the cycle is equal to area of triangle ABC.

$\therefore\ \Delta W = \frac{1}{2} \times 4 \times 5 = 10\ J$

4. **(130)** $\Delta U_{ac} = -(\Delta U_{ca}) = -(-180) = 180$ J

$Q = 250 + 60 = 310$ J

Now $Q = \Delta U + W$

or $310 = 180 + W$

or $W = 130$ J

5. **(4.263×10^3)** Heat given to water to change into steam

$Q = ML = 2.0 \times 2.3 \times 10^6$

$= 4.6 \times 10^6 J$

Volume of water $V = \frac{\text{Mass}}{\text{density}} = \frac{2.0}{10^3}$

$= 2.0 \times 10^{-3}\ m^3$

Volume of steam formed will be

$= 2.0 \times 10^{-3} \times 1671$

$= 3342 \times 10^{-3} m^3$

The change in volume in the process

$$\Delta V = V = 3342 \times 10^{-3} - 2.0 \times 10^{-3}$$
$$= 3340 \times 10^{-3} m^3$$

The work done against the atmospheric pressure

$$W = P\Delta V$$
$$= (1.01 \times 10^5) \times (3340 \times 10^{-3})$$
$$= 0.337 \times 10^6 J$$

By first law of thermodynamics

$$Q = \Delta U + W$$
$$\therefore \quad \Delta U = Q - W$$
$$= 4.6 \times 10^6 - 0.337 \times 10^6$$
$$= 4.263 \times 10^6 J.$$

Here positive value of ΔU indicates that internal energy in the process increases.

6. **(12.74)** When pressure changes from 1 atm to 3 atm , the change in pressure

$$P = 2 \text{ atm}$$
$$= 2 \times 1 \times 10^5 N/m^2$$

The force exerted on the piston

$$F = PA = 2 \times 10^5 \times 4 \times 10^{-4}$$
$$= 80 N$$

The compression of the spring

$$x = \frac{F}{k} = \frac{80}{10^4} = 0.008 m$$

The change in volume of the air due to displacement of piston by x

$$\Delta V = Ax = 4 \times 10^{-4} \times 0.008$$
$$= 3.2 \times 10^{-6} m^3$$
$$\therefore \quad \text{Final volume } V_2 = V_1 + \Delta V$$
$$= 20 \times 10^{-6} + 3.2 \times 10^{-6}$$
$$= 23.2 \times 10^{-6} m^3$$

By equation of state

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$
$$T_2 = \frac{P_2V_2T_1}{P_1V_1}$$
$$= \left(\frac{3}{1}\right) \times \frac{(23.2 \times 10^{-6})}{(20 \times 10^{-6})} \times (273 + 20)$$
$$= 1020 K$$

The change in internal energy of air

$$\Delta U = mC_V \Delta T$$
$$= (2.38 \times 10^{-5}) \times 718 \times (1020 - 293)$$
$$= 12.42 J$$

Work done in compressing the spring by x

$$W = \frac{1}{2}kx^2 = \frac{10^4}{2} \times (0.008)^2 = 0.32 J$$

From first law of thermodynamics

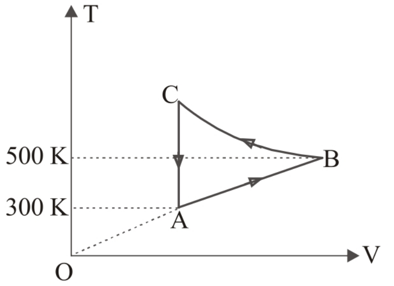
$$Q = \Delta U + W = 12.42 + 0.32 = 12.74 J.$$

7. **(4524)** In the cyclic process

$$\Delta U = 0$$

From first law of thermodynamics for the cyclic process

$$Q = \Delta U + W$$
$$\therefore \quad W = Q - \Delta U = -1200 - 0$$
$$= -1200 J$$



From C to A, $\Delta V = 0$ $\therefore$ $W_{CA} = 0$

For the whole cycle

$$W_{AB} + W_{BC} + W_{CA} = W$$
$$= -1200$$

As $W_{CA} = 0, \therefore$ $W_{AB} + W_{BC} = -1200$ J ...(i)

Work done from A to B :

In the process $V \propto T$, so pressure remains constant.

We know that $PV = nRT$

or $P\Delta V = nR\Delta T$

$\therefore$ $W_{AB} = P\Delta V = nR\Delta T = 2 \times 8.31 \times (500 - 300)$

$= 3324$ J

Substituting this value in equation (i), we get

$3324 + W_{BC} = -1200$

$\therefore W_{BC} = -4524$ J.

8. **(246.1)** Here, $P_1 = 2$ atm, $T_1 = 273 + 27 = 300$ K

When tyre burst, $P_2 = 1$ atm, $T_2 = ?$

For air $\gamma = 7/5$.

As the process is sudden, so we have

$$\frac{P_1^{\gamma-1}}{T_1^{\gamma}} = \frac{P_2^{\gamma-1}}{T_2^{\gamma}}$$

or $$\left(\frac{P_2}{P_1}\right)^{\gamma-1} = \left(\frac{T_2}{T_1}\right)^{\gamma}$$

$$\left(\frac{1}{2}\right)^{(7/5)-1} = \left(\frac{T_2}{300}\right)^{7/5}$$

After solving, we get $T_2 = 246.1$ K.

9. **(35.7)** Here, $T_2 = 273 + 27 = 300$ K

We know that $$\eta = 1 - \frac{T_2}{T_1}$$

or $$0.40 = 1 - \frac{300}{T_1}$$

or $T_1 = 500$ K

Increase in efficiency of engine = 10% of 40 = 4%

Thus new efficiency of the engine becomes = 40 + 4 = 44%

Let T_1' is the new temperature of the source, then

$$0.44 = 1 - \frac{T_2}{T_1'}$$

or $$0.44 = 1 - \frac{300}{T_1'}$$

or $T_1' = 535.7\,K$

Increase in temperature of the source

$= T_1' - T_1 = 535.7 - 500$

$= 35.7\,K.$

10. **(100)** Here, $T_1 = 273 + 100 = 373\,K$ and

$T_2 = 273 + 30 = 303\,K$

We know that $\frac{Q_1}{Q_2} = \frac{T_1}{T_2} = \frac{373}{303}$

or $Q_1 = \frac{373}{303} Q_2$ ...(i)

Work done in the cycle $W = Q_1 - Q_2 = 420\,J$ ...(ii)

From equations (i) and (ii), we have

$$\frac{373}{303} Q_2 - Q_2 = 420$$

or $Q_2 = 1818\,J$

and $Q_1 = Q_2 + 420$

$= 1818 + 420 = 2238\,J$

When the gas is carried through Carnot cycle, the heat absorbed Q_1 during isothermal expansion will equal to the work done by the gas. If V_1 and V_2 are the volumes of the gas at the beginning and at the end of the isothermal expansion, then

$$Q_1 = W = nRT\,\ell n\left(\frac{V_2}{V_1}\right)$$

or $2238 = 5 \times 8.4 \times 373\, \ell n\left(\frac{V_2}{V_1}\right)$

or $\ell n\left(\frac{V_2}{V_1}\right) = 0.1428$

or $\frac{V_1}{V_2} = \frac{115}{100}$

11. **(36.96)** Here $T_1 = 273 + 25 = 298\,K$ and

$T_2 = 273 – 15 = 258\,K$

Specific heat of water,

$C = 4.2 \times 10^3$ J/ kg K

Latent heat of fusion of ice,

$L = 3.36 \times 10^5$ J / kg

The amount of heat required to transform water of 25°C into ice of 0°C

$Q_2 = mC\Delta T + mL$

$= 2 \times 4.2 \times 10^3 \times (25 - 0) + 2 \times 3.36 \times 10^5$

$= 2.1 \times 10^5 + 6.72 \times 10^5$

$= 8.82 \times 10^5 J$

Heat rejected to the surroundings;

We have $\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$

$\therefore$ $Q_1 = Q_2\left(\frac{T_1}{T_2}\right)$

$= 8.82 \times 10^5 \left(\frac{298}{258}\right) = 10.19 \times 10^5 J$

Energy supplied to convert water into ice,

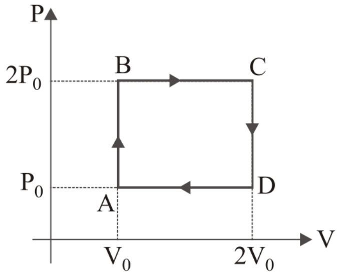
$W = Q_1 - Q_2$

$= 10.15 \times 10^5 - 8.82 \times 10^5$

$= 1.33 \times 10^5$ J

$= \frac{1.33 \times 10^5}{3600} = 36.96\ Wh.$

12. **(15.4)** If T_0 be the temperature at A, then temperature at B,



$$\frac{P_0V_0}{T_0} = \frac{2P_0 \times V_0}{T_B} \text{ or } T_B = 2T_0$$

And temperature at C becomes, $4T_0$

Heat is extracted by the system in process $A \to B$ and $B \to C$ so,

$$Q = Q_v + Q_P$$
$$= nCv\Delta T + nC_P\Delta T'$$
$$= n \times \frac{3R}{2} \times (2T_0 - T_0) + n \times \frac{5R}{2}(4T_0 - 2T_0)$$
$$= \frac{13}{2}nRT_0$$

Work done in the complete cycle

$$W = P_0V_0 = nRT_0$$

Now efficiency, $\eta = \frac{W}{Q}$

$$= \frac{nRT_0}{\frac{13}{2}nRT_0} \times 100$$
$$= 15.4\%.$$

13. **(167)** Given, $V = kT^{2/3}$

$\therefore$ $dV = k \times \frac{2}{3}T^{-1/3}dT = \frac{2}{3}kT^{-1/3}dT$

Work done $dW = PdV$

$$= \frac{RT}{V}dV$$
$$= \frac{RT}{kT^{2/3}} \times \frac{2}{3}kT^{-1/3}dT = \frac{2}{3}R(dT)$$

Total work done $W = \frac{2}{3}R\int_{T_1}^{T_2} dT$

$$= \frac{2}{3}R[T_2 - T_1]$$
$$= 2 \times 1.99 \times 30 = 39.8 \text{ cal} = 167 \text{ J}.$$

14. **(7.7)** The work done by the gas is the sum of work done W_1 against the force of atmospheric pressure and the work done W_2 against the gravity. Thus total work done,

$$W = W_1 + W_2$$

The mercury-gas interface is shifted upon the complete displacement of mercury

$$s = 2\ell + \ell/2 = \frac{5\ell}{2},$$

and hence $W_1 = Fs$

$$= P_0S \times \frac{5\ell}{2} = \frac{5P_0S\ell}{2}$$

The work done W_2 against the gravity is equal to the change in the potential energy of mercury as a result of its displacement. The whole of mercury rises as a result of displacement by l relative to the horizontal part of the tube. This quantity regarded as the final height of the centre of gravity of the whole mercury. The initial position of centre of gravity of murcury is = $\ell/8$.

Thus $$W_2 = U_2 - U_1$$

$$Mg(\ell - \ell/8) = \frac{7}{8} Mg\ell.$$

where $$M = 2\ell S\ \rho_{mer}$$

$\therefore$ $$W = W_1 + W_2$$

$$= \frac{5}{2} P_0 S\ell + \frac{7}{4}\rho_{mer} g S\ell^2$$

$$\simeq 7.7\text{ J}.$$

15. **(914.8)** Given, $T_1 = 273\ K,$

$$T_2 = 15 + 273 = 280\ K$$

Useful power $= \eta P$

$$= 0.6 \times 500 = 300\text{ J/s}$$

We know that coefficient of performance

$$\beta = \frac{Q_2}{W} = \left(\frac{T_2}{T_1 - T_2}\right)$$

$$= 300 \times \left(\frac{273}{288 - 273}\right) = 5460\text{ J/s}$$

Heat needed to melt the ice

$$Q = mL = 15 \times 333 \times 10^3\text{ J}$$

Time taken in freezing water

$$t = \frac{Q}{Q_2} = \frac{15 \times 333 \times 10^3}{5640} = 914.8\ s$$